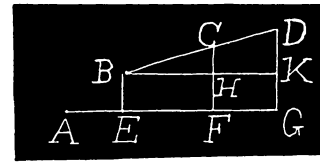
r the radius of the circle. Then $(x-b+r)^2+(y-r)^2=r^2$ is the equation to the circle. Hence the invariants are $\Delta = -4a^2$, $\Delta'=-r^2$, $\theta=-4a(b-r+a)$, $\theta'=-4a(b-r)$. In order that the circle and parabola may touch $(0\theta'-9\Delta\Delta')^2=4(\theta^2-3\Delta\theta')\times(\theta'^2-3\Delta'\theta)$.



$\therefore 16r^5+(13a-48b)r^4+8(a^2+6b^2-14ab)r^3+8(15ab^2-2b^3-9a^2b)r^2+32ab(3ab-a^2-2b^2)r+16ab^2(b^2+a^2-2ab)=0$. This gives the value of r.

Area of semi-parabola $=\frac{4}{3}b\sqrt{(ab)}$, area of circle $=\pi r^2$, area of portion left $=\frac{4}{3}b\sqrt{(ab)}-\pi r^2$.

Let A be the vertex of the parabola, B the centroid required, C the centroid of the semi-parabola, D the centroid of the circle. Let (α, β) be the coordinates of B. The coordinates of C are $(\frac{3}{5}b, \frac{3}{4}\sqrt{ab})$; of D, $(b-r, r)$.

$$\frac{CD}{BC}=\frac{HK}{BH}=\frac{GF}{EF}=\frac{\frac{4}{3}b\sqrt{(ab)}-\pi r^2}{\pi r^2} \text{ or } \frac{EG}{EF}=\frac{4b\sqrt{(ab)}}{3\pi r^2}.$$

$$\therefore \frac{b-r-\alpha}{\frac{3}{5}b-\alpha}=\frac{4b\sqrt{(ab)}}{3\pi r^2}. \quad \therefore \alpha=\frac{3[4b^2\sqrt{(ab)}+5\pi r^3-5\pi br^2]}{5[4b\sqrt{(ab)}-3\pi r^2]}.$$

$$\text{And } \frac{DK}{CH}=\frac{EG}{EF}=\frac{4b\sqrt{(ab)}}{3\pi r^2}=\frac{r-\beta}{\frac{3}{4}\sqrt{(ab)}-\beta}; \quad \therefore \beta=\frac{3(ab^2-\pi r^3)}{4b\sqrt{(ab)}-3\pi r^2}.$$

208. Proposed by W. J. GREENSTREET, M. A., Marling School, Stroud, Eng.

Hanging at rest over a smooth pulley are two equal scale pans of the same mass. Two equal particles, the one inelastic and the other elastic, are simultaneously dropped from the same height one into each scale pan. Show that each impact after the first must occur when the pans have returned to the *status quo ante*, and find the total space described by either pan before motion ceases.

Remark by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

This is the same as problem 121, Mechanics. A solution of this problem is found in Vol. VIII, No. 10, pp. 203-4, October, 1901.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

141. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Given that the highest factor of a prime p contained in $m!$ is p^{m-s}; find general expressions involving p and m and s, from which, when a solution is possible, m can be determined when s is a given integer and p is a given prime. Is it then possible in any case to have more solutions than one?